Lecture 11: Boosting

STATS 202: Data Mining and Analysis

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Announcements

- Kaggle predictions due in 11 days.
- Homework 4 is out (due in 9 days)
- Final exam details TBD
- Final project review this Friday.

Outline

- Boosting introduction
- Boosting vs bagging
- Boosting remarks
- AdaBoost
- Boosting training error
- Gradient boosting
- Regularization
- Random tips

Recall

- Decision trees partition our feature space and make predictions within each partitioned region.
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Question: Is there another way of improving the performance of decision trees?

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3. Output prediction, e.g.
   $$\hat{f}_n(x) = \sum_{b=1}^{B} \lambda_b \hat{f}_n^b(x). \tag{3}$$

Boosting hyperparameters

Hyper-parameters to consider when applying a boosting model:

- The number of learners (aka trees) B to use.
- The shrinkage parameter λ_b.
- The parameters of the learner (e.g. splits in each tree).

Typically, these are found via *cross-validation*.

Boosting vs bagging

Bagging: For $b = 1, \ldots, B$:

1. Created a bootstrapped sample, P_n^b.
2. Get estimate $\hat{f}_n^b(x)$ using P_n^b.

Average the estimates, i.e.

$$\hat{f}_n^{\text{bag}}(x) = \frac{1}{B} \sum_{b=1}^{B} \hat{f}_n^b(x).$$

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Boosting: For $b = 1, \ldots, B$:

1. Get estimate $\hat{f}_n^b(x)$ for the residuals r^{b-1}.
2. Update residuals $r_i^b = r_i^{b-1} - \lambda_b \hat{f}_n^b(x_i)$.

Sum the estimates, i.e.

$$\hat{f}_n^{\text{boost}}(x) = \sum_{b=1}^{B} \lambda_b \hat{f}_n^b(x).$$

- 'Y' is varied for each fit.
- Designed to reduce bias.

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- Boosting can give zero training error, but rarely overfits.
- Can be thought of as fitting a model on multiple data sets.

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$$err_b = \frac{\sum_{i=1}^{n} w_i \mathbb{I}(y_i \neq G^b(x_i))}{\sum_{i=1}^{n} w_i} \tag{4}$$

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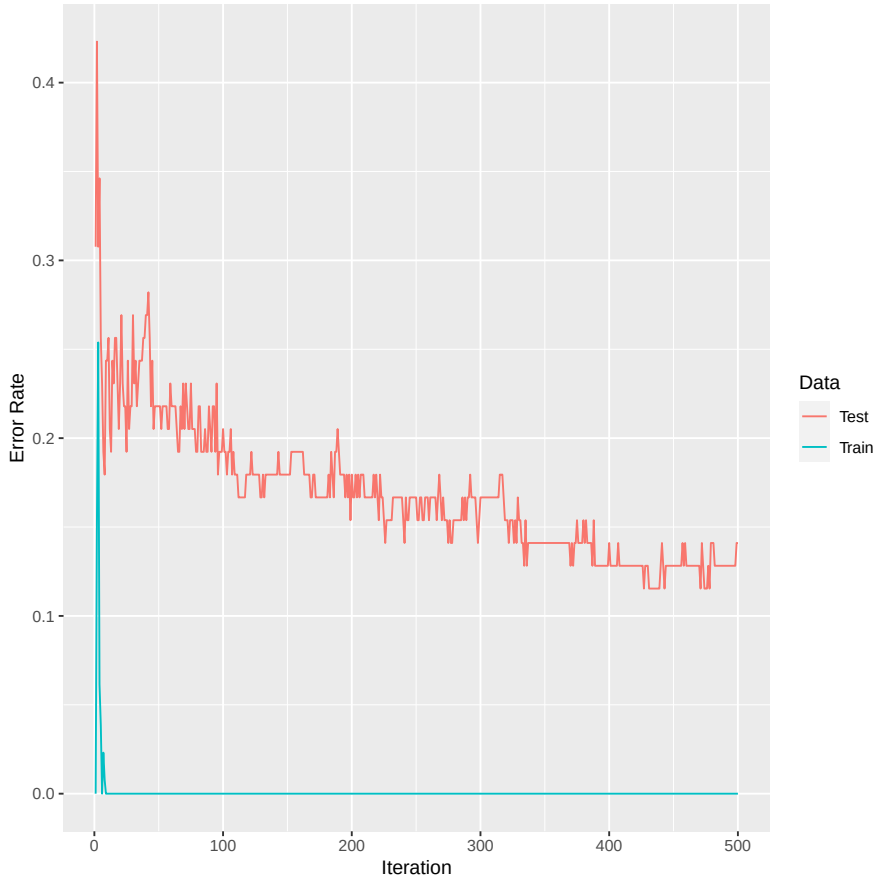
3. Output $G_B(x) = \text{sign}\left(\sum_{b=1}^B \lambda_b G^b(x)\right)$.

AdaBoost example



AdaBoost applied to the *Sonar Data*.

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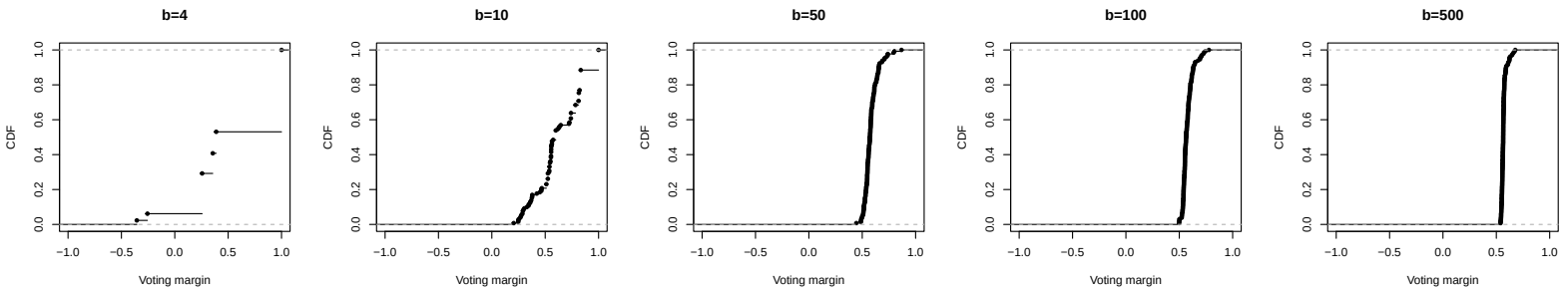
We can look at *voting margins* for our training data, i.e.

$$margin(x) = y * G_B^*(x) \tag{6}$$



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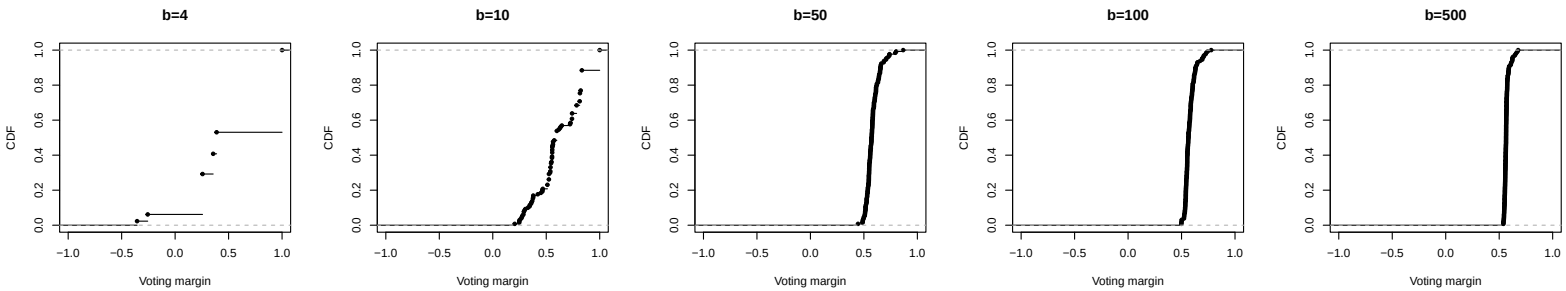

n.b. Letting $err_b \leq 1/2 - \gamma$, then $\text{Error}_{train} \leq (\sqrt{1 - 4\gamma^2})^B$

Gradient Boosting

AdaBoost can be framed as Forward Stagewise Additive Modeling:

Algorithm 10.2 *Forward Stagewise Additive Modeling.*

1. Initialize $f_0(x) = 0$.
2. For $m = 1$ to M:
   (a) Compute
   $$(\beta_m, \gamma_m) = \arg\min_{\beta,\gamma} \sum_{i=1}^{N} L(y_i, f_{m-1}(x_i) + \beta b(x_i; \gamma)).$$
   (b) Set $f_m(x) = f_{m-1}(x) + \beta_m b(x; \gamma_m)$.

where $L(y, f_{m-1}(x) + \beta b(x; \gamma))$ is the exponential loss, i.e.

$$L(y, f(x)) = \exp(-yf(x)) \tag{7}$$

Gradient Boosting

Gradient boosting generalizes $L(y, f(x))$ to any smooth loss function.

Some common loss functions:

TABLE 10.2. *Gradients for commonly used loss functions.*

Setting	Loss Function	$-\partial L(y_i, f(x_i))/\partial f(x_i)$
Regression	$\frac{1}{2}[y_i - f(x_i)]^2$	$y_i - f(x_i)$
Regression	$\lvert y_i - f(x_i)\rvert$	$\text{sign}[y_i - f(x_i)]$
Regression	Huber	$y_i - f(x_i)$ for $\lvert y_i - f(x_i)\rvert \leq \delta_m$ $\delta_m \text{sign}[y_i - f(x_i)]$ for $\lvert y_i - f(x_i)\rvert > \delta_m$ where $\delta_m = \alpha\text{th-quantile}\{\lvert y_i - f(x_i)\rvert\}$
Classification	Deviance	kth component: $I(y_i = \mathcal{G}_k) - p_k(x_i)$

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$$\gamma_i^b = \arg\min_{\gamma} \sum_{x_i \in R_j^b} L(y_i, f^{b-1}(x_i) + \gamma) \tag{9}$$

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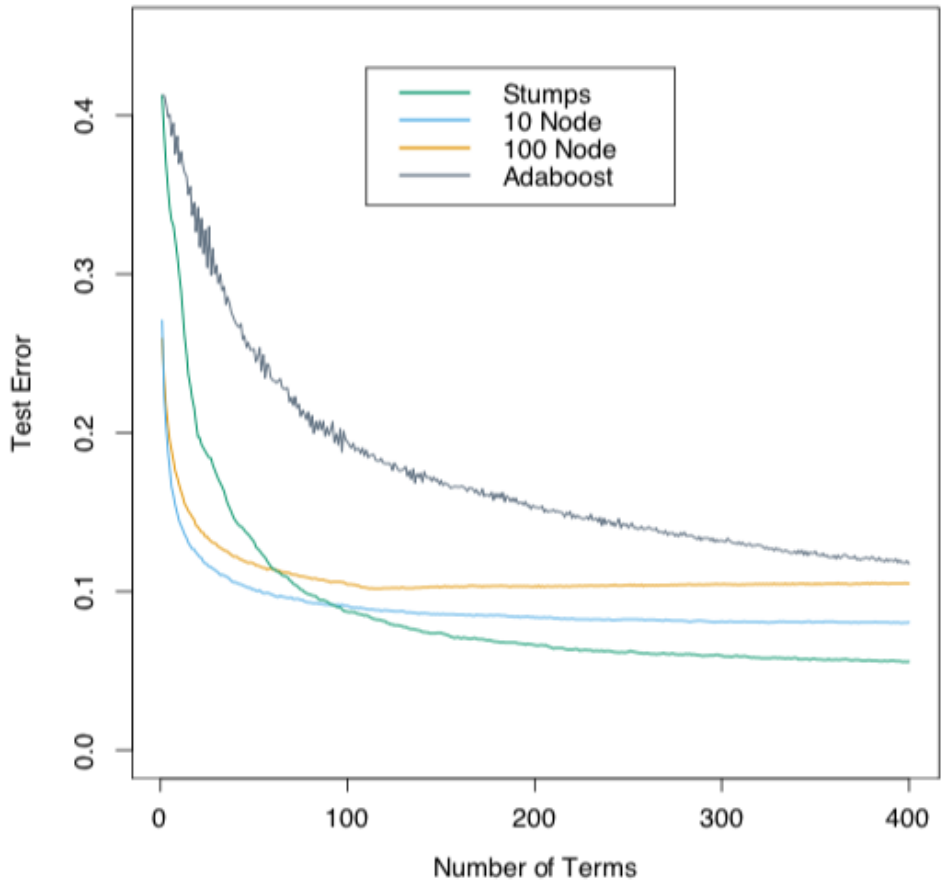
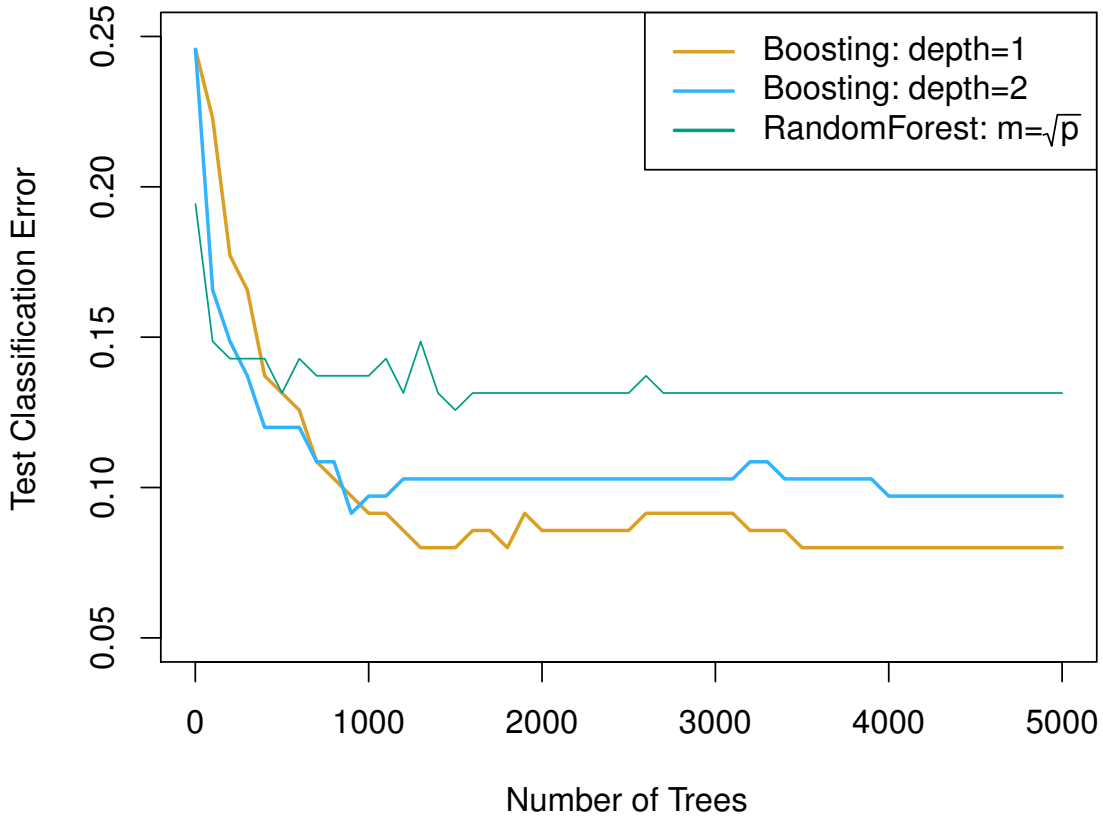
Boosting vs. AdaBoost

Example: Applied to simulated data.



Boosting vs. random forests

Example: Applied to 15-class gene expression data.



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- Penalized learning: Apply $L1$ or $L2$ regularization to the terminal nodes.

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Some tips from Kaggle master *Owen Zhang*:

GBDT Hyper Parameter Tuning

Hyper Parameter	Tuning Approach	Range	Note
# of Trees	Fixed value	100-1000	Depending on datasize
Learning Rate	Fixed => Fine Tune	[2 - 10] / # of Trees	Depending on # trees
Row Sampling	Grid Search	[.5, .75, 1.0]	
Column Sampling	Grid Search	[.4, .6, .8, 1.0]	
Min Leaf Weight	Fixed => Fine Tune	3/(% of rare events)	Rule of thumb
Max Tree Depth	Grid Search	[4, 6, 8, 10]	
Min Split Gain	Fixed	0	Keep it 0

Best GBDT implementation today: https://github.com/tqchen/xgboost
by **Tianqi Chen** (U of Washington)

DataRobot

References

[1] ISL. Chapter 8

[2] ESL. Chapter 10

[3] Schapire, RE. The Boosting Approach to Machine Learning An Overview. Nonlinear Estimation and Classification, Springer, 2003.