STATS 202: Data Mining and Analysis

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HOMEWORK # 4

Due date: August 11, 2023

Stanford University

Introduction

Homework problems are selected from the course textbook: *An Introduction to Statistical Learning*.

Problem 1 (10 points)

Chapter 8, Exercise 4 (p. 362).

Problem 2 (10 points)

Chapter 8, Exercise 8 (p. 363).

Problem 3 (10 points)

Chapter 8, Exercise 10 (p. 364).

Problem 4 (10 points)

Chapter 10, Exercise 3 (p. 459).

Problem 5 (Bonus 10 points)

Let $x_i : i = 1, ..., p$ be the input predictor values and $a_k^{(2s)} : k = 1, ..., K$ be the K-dimensional output from a 2-layer and M-hidden unit neural network with sigmoid activation $\sigma(a) = \{1 + e^{-a}\}^{-1}$ such that

$$
\begin{aligned}
a_j^{(1s)} &= w_{j0}^{(1s)} + \sum_{i=1}^{p} w_{ji}^{(1s)} x_i : j = 1, ..., M \\
a_k^{(2s)} &= w_{k0}^{(2s)} + \sum_{j=1}^{M} w_{kj}^{(2s)} \sigma\left(a_j^{(1s)}\right)
\end{aligned}
$$

Show that there exists an equivalent network that computes exactly the same output values, but with hidden unit activation functions given by $tanh(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$, i.e.

$$
\begin{aligned}
a_j^{(1t)} &= w_{j0}^{(1t)} + \sum_{i=1}^{p} w_{ji}^{(1t)} x_i : j = 1, ..., M \\
a_k^{(2t)} &= w_{k0}^{(2t)} + \sum_{j=1}^{M} w_{kj}^{(2t)} \tanh\left(a_j^{(1t)}\right)
\end{aligned}
$$

Hint: first derive the relation between $\sigma(a)$ and $tanh(a)$. Then show that the parameters of the two networks differ by linear transformations.